**Announcements**

- **Reminder**: The first problem set (assigned last week) is due next Monday, February 9, at the start of class.

**Reading Guide**

This week we read about Newtonian cosmology and derive the fundamental relations that will allow us to discuss the dynamic evolution of the Universe. This will prepare us for a tour of the early universe in anticipation of our study of relativistic cosmology.

1. **Text – Chapter 14, Section 1 (partial): The High Cosmological Redshifts of Quasars.**

   This reading should have been assigned last week, since that’s when we introduced the concept of the scale factor. So, read here just the short, 1 page subsection of §14.1, on pages 590 – 591 of your text, in which the meaning of the cosmic scale factor is fleshed out.

2. **Text – Chapter 15, Section 1: Newtonian Cosmology.**

   This section covers what we did in class this week, and sets up nearly everything we shall cover later on, so read carefully!

3. **Optional Reading: Ryden, §§2.2, 2.3, 4.1, 4.2, 4.3, and 5.1.**

   The recommended text, *Introduction to Cosmology*, by Barbara Ryden, covers material in a somewhat different order than your text. Most notably, it mixes in a discussion of general relativity right from the start, whereas Carroll & Ostlie holds off until later. Thus, there is not a direct, linear correspondence of sections between the two books. A most fruitful endeavor will be to simply read Ryden’s book straight through at the end of our section on cosmology. I believe that you will find Ryden’s exposition to provide additional insight beyond that provided by Carroll & Ostlie, as she works out the consequences of different cosmological parameters in more detail than your text.

   Before then, though, reading this recommended text as we proceed can also be very useful, so long as you are careful and recognize that the presentations do differ. To aid this approach, I will try to pick out those sections each week that bear a direct relation to the required reading, thereby sacrificing linearity for correspondence of subject matter. If you wish, of course, you may simply read Ryden straight through (up to §5.1, for this week), and you’ll be ahead of the game!

**Homework Questions for Problem Set 2: Due Monday, February 16**

The following questions represent the first part of the second homework set, which you will turn in at the start of class on **Monday, February 16**.

Please answer the following questions as completely as possible. In the case of numerical problems, **please indicate your final answer by circling it.** Partial credit for incorrect answers will only be given if work is clearly shown.

1. (20 points total). For a one-component universe of pressureless dust:

   (a) (10 points) Make a plot that shows how \( \frac{H}{H_0} \) varies with redshift for \( 0 \leq z \leq 5 \), and \( \Omega_0 = 0, 0.5, 1.0, \) and \( 2.0 \). Put all four of the curves on the same plot, and clearly label them.

   (b) (10 points) Describe in words as clearly as you can how the value of \( \frac{H}{H_0} \) for the \( \Omega_0 = 0 \) (“empty”, or “Milne” Universe) makes sense at the time in the Universe’s evolution corresponding to redshift \( z = 1 \).
2. (10 points). For a one-component universe of pressureless dust, show that

\[ \frac{1}{\Omega} - 1 = \left( \frac{1}{\Omega_0} - 1 \right) (1 + z)^{-1}. \]

To derive this equation, you may use any of the equations given to you in Section 15.1 of the text as starting points (i.e., you do not have to derive the text equations, just use them to derive the equation above). Discuss the behavior of \( \Omega \) as \( z \to \infty \).

3. (10 points) For the special case where \( P = 0 \), the acceleration equation (Eq. 15.51) reduces to a very simple form. Show that this form can be derived by simply applying Newton’s second law to the gravitational force on an expanding mass shell in a one-component universe of pressureless dust.

4. (10 points). Derive the functional form for \( q_0 \):

\[ q_0 = -\left( \frac{\ddot{R}R}{R^2} \right) \]

To do this, begin by writing the Taylor expansion for \( R(t) \) about the present epoch, \( t = t_0 \), including only the first three terms of the expansion. Then, express the terms of the Taylor expansion in terms of powers of \( H_0 \), and show that the parameterization of \( q_0 \) given above is required for the third term of the series.

5. (20 points total) Consider two possible universes (U1 and U2) where the scale factor varies with time as \( R(t) = \gamma t^{2/3} \) and \( R(t) = \alpha e^{\beta t} \), respectively. Answer the following two questions for each universe.

(a) (10 points) How does the Hubble parameter, \( H(t) \), depend on time?

(b) (10 points) What is the value of the deceleration parameter, \( q_0 \), and in what dynamical state (i.e., accelerating, decelerating, coasting) is the Universe in?

6. (15 points) A long time ago in a galaxy far, far away, an alien astronomer observes a quasar and measures its redshift. The alien astronomer then immediately encodes this information (i.e., the redshift of the quasar that she just observed) in a super-high powered laser pulse and fires it in the direction 180° opposite the quasar, which happens to be right at the Milky Way. One night many, many years later, astronomer Rohit on Earth is observing both the alien astronomer’s galaxy and the very same quasar that was observed by her (they lie effectively along the same line-of-sight). He determines their redshifts to be \( z = 1 \) (galaxy) and \( z = 2 \) (quasar). Just as he is observing the galaxy, he is startled to receive the laser pulse that was sent by the alien astronomer. When he decodes her message, it says: “I measure the redshift of the quasar to be...” [complete the alien astronomer’s sentence].

7. (30 points total) Consider a critical (i.e., flat) universe that is filled only with radiation (i.e., no matter or any other component, at all times), with the spectrum of a blackbody corresponding to \( T_0 = 2.725 \) K at the present time.

(a) (3 points) What is the present energy density of this universe? Express your answer in J m\(^{-3}\).

(b) (8 points) What is the present value of the Hubble constant? State your answer first in s\(^{-1}\), and then convert it to units of km s\(^{-1}\) Mpc\(^{-1}\). (Hint: Remember again that this is a critical universe, and that the only component contributing to the energy density is radiation!)

(c) Let’s now determine some interesting things about how this universe ages.

i. (10 points) Derive a general expression for the age of this universe as a function of the Hubble time, \( \frac{1}{H} \).

ii. (2 points) How old is this universe at the present time? Express your answer in years.

iii. (7 points) Derive a general expression for the age of this universe as a function of the temperature of the radiation, \( T \). You may leave your answer expressed in terms of fundamental constants \( c, G, \) and \( a \).