Here are solutions for the problems assigned on Weekly Handouts #2, #3, and #4.

**Homework for Week #2**

1. Page 40, *Voyages to the Stars and Galaxies*, “Review Question” #1: From where on Earth could you observe all of the stars during the course of a year? What fraction of the sky can be seen from the North Pole?

   **Solution:** At any point on Earth, half of the celestial sphere is viewable at any given time. However, from the North Pole, it is the same half that is always visible throughout the year (and at any time during any given day); the viewable portion of the sky does not change, and an observer located there will always see the Northern half of the celestial sphere. Similarly, at the South Pole, an observer would only see the Southern half of the celestial sphere. On the other hand, at the equator, the Earth’s revolution around the sun has the effect of moving the sun 180° in the sky during each six-month period. This results in a complete change of the stars that are viewable at night: the half of the celestial sphere that is overhead at midnight on, say, January 1, is diametrically opposite to the half of the celestial sphere that is overhead at midnight exactly six months later (midnight, July 1). Thus:

   - All of the stars are visible during the course of a year from the equator.
   - Only half of the sky is visible from the North Pole.

2. Page 41, *Voyages to the Stars and Galaxies*, “Figuring for Yourself” Question #19, part (c) only. Note that for this question you may assume that the “strange planet” you are on is spherical.

   Suppose you are on a strange planet and observe, at night, that the stars do not rise and set but circle parallel to the horizon. Now you walk in a constant direction for 8000 miles, and at your new location on the planet you find that all stars rise straight up in the east and set straight down in the west, perpendicular to the horizon. What is the circumference, in miles, of that planet?

   **Solution:** Since stars circle parallel to the horizon at the poles, but rise and set straight up and down at the equator, it is clear that by walking 8000 miles you have walked from a pole (either North or South) to the equator of this planet. Since this distance is 1/4 of the complete way around the spherical planet, the circumference of the planet is just:

   \[ 4 \times 8000 \text{ miles} = 32,000 \text{ miles} \]

   The planet thus has a circumference of 32,000 miles.

**Homework for Week #3**

1. Page 59, “Figuring for Yourself” question #23. By what factor would a person’s weight at the surface of the Earth be reduced if the Earth had its present mass but eight times its present volume? What if it had its present size but only one-third its present mass?

   **Hint:** This problem is a toughie, and really tests your ability to use ratios (i.e., did you read the Mathematical Toolkit handout? If not, read it NOW, and pay particular attention to the example problem concerning volumes!)

   **Solution:** There are really two approaches to solving this problem. The first involves thinking carefully about what the problem is telling you and then solving for the answer. The second involves formally writing down all of the needed equations and solving for the answer. The second approach will always get you the right answer, but the first approach is much quicker! So, let’s try both ways, tackling the first of the two posed question first.

   **Method 1.** Thought process: For this problem, the mass of the person and the mass of the Earth remain unchanged; only the radius of the planet is changing. Since the volume of a sphere increases
as the cube of its radius, the new radius will be $\sqrt[3]{8} = 2$ times its present radius. Since force (i.e., weight) at the surface of a planet decreases as the square of the radius, the weight would be decreased to $\frac{1}{4}$ its present value. Therefore the person’s new weight is only 1/4 her old weight.

**Method 2.** OK, let’s write down all of the needed equations, using the subscripts $B$ and $A$ to represent ‘before’ and ‘after’, respectively; $p$ and $\oplus$ to represent the ‘person’ and the ‘Earth’, respectively; and $W$ and $V$ to represent the ‘weight’ and ‘volume’, respectively.

We know:

$$W = \frac{GM_{\oplus}M_p}{R_{\oplus}^2}$$

and we are trying to solve for:

$$\frac{W_A}{W_B}$$

So:

$$W_B = \frac{GM_{\oplus,B}M_{p,B}}{R_{\oplus,B}^2}$$

$$W_A = \frac{GM_{\oplus,A}M_{p,A}}{R_{\oplus,A}^2}$$

Now, $M_p$ and $M_{\oplus}$ do not change, so $M_{\oplus,A} = M_{\oplus,B} = M_{\oplus}$ and $M_{p,A} = M_{p,B} = M_p$.

$$\frac{W_A}{W_B} = \frac{GM_{\oplus}M_p}{GM_{\oplus,B}M_p} \times \frac{R_{\oplus,A}^2}{R_{\oplus,B}^2}$$

We can simplify this complex fraction by multiplying the top and bottom fractions by the reciprocal of the bottom fraction to yield:

$$\frac{W_A}{W_B} = \frac{GM_{\oplus}M_p}{GM_{\oplus,B}M_p} \times \frac{R_{\oplus,B}^2}{R_{\oplus,A}^2}$$

Canceling the terms in common to the top and bottom yields:

$$\frac{W_A}{W_B} = \left(\frac{R_{\oplus,B}}{R_{\oplus,A}}\right)^2$$

(1)

OK! We now have to use the given information to determine what the ratio $R_{\oplus,B}/R_{\oplus,A}$ is equal to. We do this by considering the information about the change in volume. We are given:

$$\frac{V_A}{V_B} = 8$$

And we know the formula for volume of a sphere:

$$V = \frac{4}{3}\pi R^3$$

Thus we have:

$$V_B = \frac{4}{3}\pi R_{\oplus,B}^3$$
and

\[ V_A = \frac{4}{3} \pi R_{\oplus,A}^3 \]

So that we have:

\[ 8 = \frac{\frac{4}{3} \pi R_{\oplus,A}^3}{\frac{3}{\pi} R_{\oplus,B}^3} \]

Canceling the common factors in the top and bottom then yields:

\[ 8 = \frac{R_{\oplus,A}^3}{R_{\oplus,B}^3} \]

Taking the cube root of both sides

\[ 3\sqrt{8} = \sqrt[3]{\frac{R_{\oplus,A}^3}{R_{\oplus,B}^3}} \]

yields:

\[ 2 = \frac{R_{\oplus,A}}{R_{\oplus,B}} \]

Thus,

\[ \frac{R_{\oplus,B}}{R_{\oplus,A}} = \frac{1}{2} \]

Substituting equation (2) into equation (1) yields:

\[ \frac{W_A}{W_B} = (\frac{1}{2})^2 \]

Or:

\[ \frac{W_A}{W_B} = \frac{1}{4} \]

The person will weigh only 1/4 as much as she did before when the Earth’s volume is increased by a factor of 8.

OK – so, note that either approach is acceptable to arrive at the final answer, but the first one takes far less time!

Let’s now answer the second question: What if it (the Earth) had its present size but only one-third its present mass?

We can again solve this using the two methods.

Method 1. Thought process: Force on the person (i.e., her weight) increases linearly with mass. Therefore, if the Earth’s mass decreases to 1/3 of its present value, the person’s weight will also decrease to 1/3 of its present value.

Method 2. Using the same thought process as for the first part (but now it’s the Earth’s mass that is changing):
\[ \frac{W_A}{W_B} = \frac{GM_{B,A}M_p}{R_B^3} \]
\[ \frac{W_A}{W_B} = \frac{M_{B,A}}{M_{B,B}} \]

Since we are given that \( \frac{M_{B,A}}{M_{B,B}} = 1/3 \) we have:

\[ \frac{W_A}{W_B} = \frac{1}{3} \]

The person’s weight will be only 1/3 of her former weight.

**Homework for Week #4**

1. p. 106, thought question #15. The planet Jupiter appears yellow and Mars is red. Does this mean that Mars is cooler than Jupiter? Explain your answer.

   **Hint:** Recall that planets shine primarily due to reflected light.

   **Solution:** The visible light we see from both Jupiter and Mars is primarily reflected sunlight, not emission from a hot gas (like a star) or a glowing, hot, opaque object. Thus, their colors represent the reflective properties of their surfaces (or clouds) and are unrelated to their temperatures. The actual surface temperature of Mars ranges from about 150 K to 310 K, which is, in fact, somewhat warmer than the mean temperature of about 130 K at the cloud tops in Jupiter’s atmosphere. (Since Jupiter has no solid surface, it is difficult to define a “surface temperature”.) Thus, Mars is in fact somewhat warmer than Jupiter; this is largely due to its much closer distance to the Sun.

2. p. 107, figuring for yourself, EITHER #23 OR #24.

   (a) p. 107 #23: How many times brighter or fainter would a star appear if it were moved to:

   i. twice its present distance.

      **Solution:** This is a question that concerns the inverse square law of light, as discussed in section 4.1.4: The brightness of a light source decreases as the square of its distance from us \((1/\text{distance}^2)\). Thus, at twice the distance the star appears \[ \text{four times fainter} \]

   ii. ten times its present distance.

      **Solution:** \[ \text{100 times fainter} \]

   iii. half its present distance.

      **Solution:** \[ \text{4 times brighter} \]

   (b) p. 107 #24: Two stars with identical diameters are the same distance away. One has a temperature of 5800 K; the other has a temperature of 2900 K. Which is brighter? How much brighter is it?

      **Solution:** This problem concerns the Stefan-Boltzmann law (p. 93): \( F = \sigma T^4 \), which gives the total energy emitted per second per square meter by a star. The two stars of this problem have the same diameter and therefore the same surface area in square meters. The one that is twice as hot thus emits \( (2)^4 \) or 16 times more energy than the cooler one. Thus, the hotter star appears 16 times brighter than the cooler star.