# Johannes Kepler (1571 - 1630)

(the first astrophysicist)



(Fraknoi,Morrison, & Wolff: *Voyages to the Stars and Galaxies,* 3rd Edition, Figure 2.2, Page 44)

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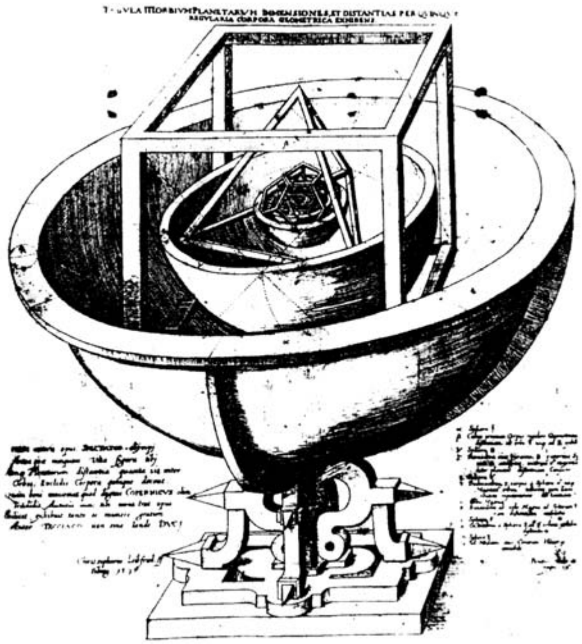
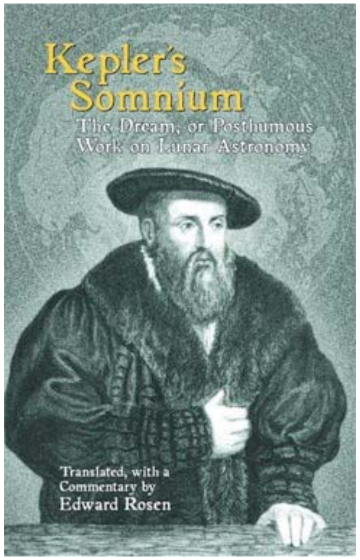
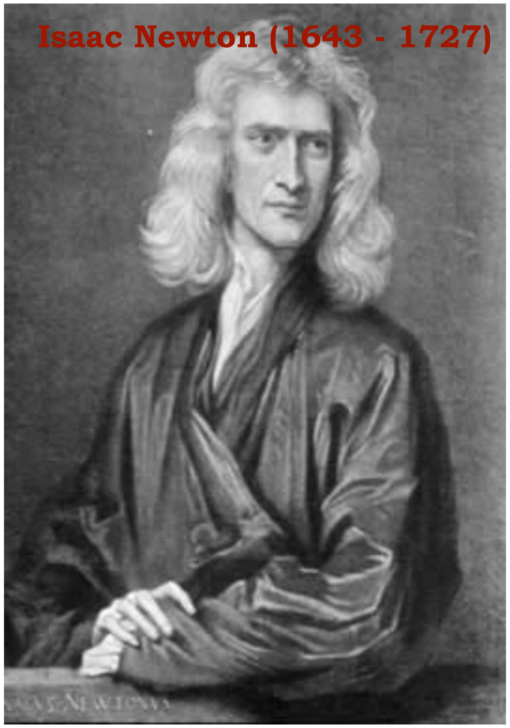
(Fraknoi,Morrison, & Wolff: *Voyages to the Stars and Galaxies,* 3rd Edition, Figure 2.2, Page 44)






## Two Fundamental Questions about the Planets



**What** are the precise paths taken by the planets as they revolve around the Sun?

**Johannes Kepler (1571 - 1630)**



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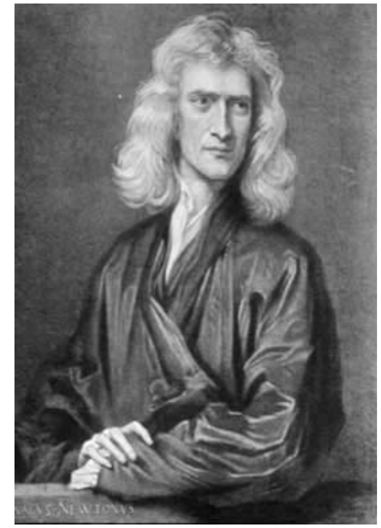
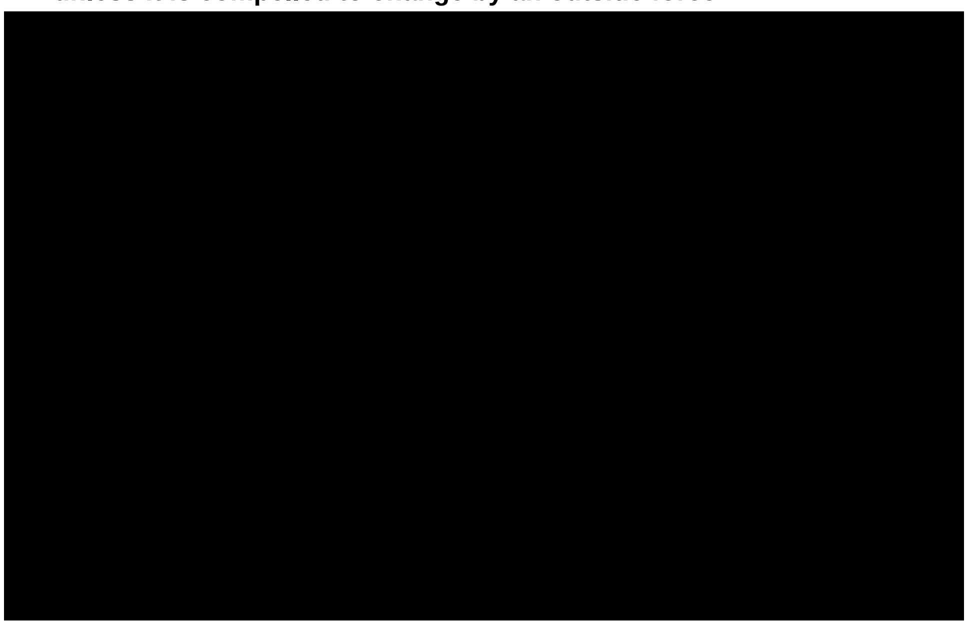
**Why** do the planets follow the paths that they do?

**Isaac Newton (1643 - 1727)**



**Law of Inertia**: **Every body tends to continue doing what it is already doing -- being in a state of rest, or moving uniformly in a straight line -- unless it is compelled to change by an outside force.**



## Newton's Law of Gravitation

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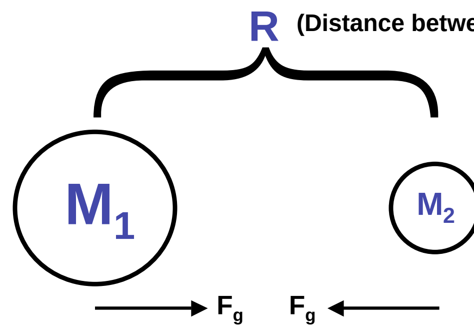


$$F_g = \frac{GM_1M_2}{R^2}$$

$M_1$ = Mass of object 1
$M_2$ = Mass of object 2
R = Distance between the CENTERS of the two objects
G = Gravitational constant

## Fundamental Forces of Nature

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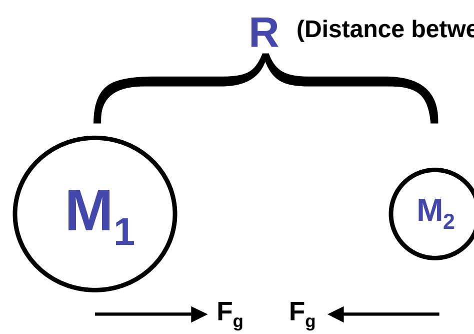
The basic forces that are known to exist in Nature:

- **Gravity**

## Newton's Law of Gravitation

84

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# Using Newton's Law of Gravity

**86**

**Suppose the Earth was located twice as far from the Sun as it currently is. How would the gravitational force exerted by the Sun on the Earth at this new distance compare with its strength at its old distance?**

$$F_g = \frac{GM_1M_2}{R^2}$$

## The Mathematical Toolkit

The third link, "figures and polygons" goes into some detail on a lot of different types of figures. It turns out that for astronomy, the most important shape by far is the circle. Thus, pay particular attention to the review of terms for the circle (e.g., the radius $r$). In the next section, "area and perimeter", focus again on the circle, as you will be expected to know these relations:

- Circumference of circle = $2\pi r$.
- Area of circle = $\pi r^2$.

In the next section, "Coordinates and similar figures", you need only review the first section, entitled "What is a Coordinate?". This provides a very important review of *graphing* techniques, useful throughout the course. In the final section of the web page, entitled "Space figures and basic solids", focus attention particularly on the *sphere*, for which:

- Surface area of sphere = $4\pi r^2$.
- Volume of sphere = $\frac{4}{3}\pi r^3$.

Again, you are responsible for knowing these important relationships. Note that the key thing to remember in all of these formulas (for circles and spheres) is the *power* to which the radius $r$ is raised. This is what is really needed when determining *ratios* involving circles and spheres, which is of utmost importance for this course, since, in general, *ratios* are what you will be asked to solve. What this means is that the most important part of the above equations is the $r$ part, NOT the constant numbers out in front (e.g., $4\pi$, etc.). So, focus on the variable, not the constants. Indeed, all equations can be expressed as *proportionalities* (indicated by the mathematical symbol $\propto$) instead of equalities by eliminating the constants; for instance, we can rewrite the formula for volume of a sphere as a proportionality by eliminating the $\frac{4}{3}\pi$ part:

$$\text{Volume of sphere} \propto r^3$$

That is, the volume of a sphere increases in proportion to the cube of its radius. Thinking of the formulas as proportionalities is very helpful when trying to determine ratios, since it forces you to consider only the relevant part of the equation.

Since using ratios is such an important part of this course, I've worked some examples for you in the next section, just to make sure you've gotten it!

### Working with Ratios

There are essentially two techniques that can be used to solve a problem involving ratios: The "long way" (Method 1) and the "short way" (Method 2). The advantages of Method 1 are that it will *always* produce the correct answer and does not really require you to *think* about what you are doing; the disadvantage is that it can take a long time to work out! The main advantage of Method 2 is that it results in a very fast answer; a disadvantage (although some may see it as an advantage, actually) is that you do have to *think* about the nature of the problem at hand in order to arrive at the correct answer.

Let's work through an example so that you can see both "methods" at work. Note that all such problems can be done *without a calculator*; indeed, you will not have access to a calculator during any exams, so it is important to get used to carrying out these sorts of problems on paper!

*Example Problem:* Jupiter's radius is roughly 10 times that of the Earth, and you may assume that both Jupiter and Earth are perfect spheres.

1. What is the ratio of Jupiter's *volume* to that of the Earth?

**Solution using Method 1 ("The Long Way"):** Solving this problem using Method 1 involves the following 6 steps:

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*Step 1: Write down the relevant formula:* Here, the relevant formula is the volume of a sphere:

$$\text{volume of sphere} = \frac{4}{3}\pi r^3$$

*Step 2: Write down mathematically all information stated in the problem.* In this problem, we are told that the radius of Jupiter is about 10 times the radius of the Earth. Expressing this mathematically (calling Earth's radius $r_E$ and Jupiter's radius $r_J$):

$$r_J = 10r_E$$

*Step 3: Set up the mathematical ratio to be solved.* The ratio of Jupiter's volume to Earth's volume is:

$$\frac{\text{volume of Jupiter}}{\text{volume of Earth}} = \frac{\frac{4}{3}\pi r_J^3}{\frac{4}{3}\pi r_E^3}$$

*Step 4: Cancel all like terms from the numerator (top) and denominator (bottom) of the ratio.* Notice that the $\frac{4}{3}\pi$ cancels from the top and the bottom of the equation, which leaves just:

$$\frac{\text{volume of Jupiter}}{\text{volume of Earth}} = \frac{r_J^3}{r_E^3}$$

*Step 5: Insert the relationship from Step 2 into the equation derived in Step 4.* Since $r_J = 10r_E$, we can substitute $10r_E$ for $r_J$ in the equation written in step 4, which leaves us with:

$$\frac{\text{volume of Jupiter}}{\text{volume of Earth}} = \frac{(10r_E)^3}{r_E^3}$$

*Step 6: Cancel like terms in the numerator and denominator of Step 5, and solve the problem.* Since $(10r_E)^3 = 10^3 r_E^3$, we have:

$$\frac{\text{volume of Jupiter}}{\text{volume of Earth}} = \frac{10^3 r_E^3}{r_E^3}$$

Canceling the $r_E^3$ from top and bottom leaves us with:

$$\frac{\text{volume of Jupiter}}{\text{volume of Earth}} = \frac{10^3}{1}$$

Since $10^3 = 1000$, we thus conclude that:

$$\boxed{\frac{\text{volume of Jupiter}}{\text{volume of Earth}} = 1000}$$

That is, Jupiter's volume is 1000 times that of the Earth! Note that at no time did this problem require us to know the *actual* value of Earth's (or Jupiter's) radius. All we needed was the *ratio* of the radii.

**Solution using Method 2 ("The Short Way"):** Solving this problem using Method 2 involves the following 4 steps (almost as many steps as Method 1, but much quicker to carry out):

*Step 1: Write down the two quantities to be compared, and decide which one will be bigger.* This is the part of "The Short Way" that requires you to think about the question at hand. Here, we are comparing the volume of Jupiter to the volume of Earth. Since Jupiter is a larger planet, it makes sense that its volume will be greater than that of the Earth. Thus, we know:

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## Working With Ratios: "The Short Way" (from Mathematical Toolkit)

**Solution using Method 2 ("The Short Way"):** Solving this problem using Method 2 involves the following 4 steps (almost as many steps as Method 1, but much quicker to carry out):

→ *Step 1: Write down the two quantities to be compared, and decide which one will be bigger.* This is the part of "The Short Way" that requires you to think about the question at hand. Here, we are comparing the volume of Jupiter to the volume of Earth. Since Jupiter is a larger planet, it makes sense that its volume will be greater than that of the Earth. Thus, we know:

$$\text{volume of Jupiter} > \text{volume of Earth}$$

→ *Step 2: Write down the relevant formula as a proportionality by eliminating all constants (including variables that do not change).* Here, the relevant formula is the volume of a sphere, which is $\frac{4}{3}\pi r^3$; eliminating the constants gives:

$$\text{volume} \propto r^3$$

→ *Step 3: Replace the variable(s) in Step 2 with the factor(s) by which the variable(s) is changing between the two situations being compared.* Here, we are given that $r_J = 10r_E$; thus, the relevant factor is the number 10. So:

$$\text{Ratio of volumes} = 10^3 = 1000$$

This tells us that the two planets' volumes differ by a factor of 1000.

→ *Step 4: Use your answers from Steps 1 and 3 to answer the original question.* OK, from Step 1 we know that Jupiter's volume is greater than Earth's; from Step 3 we know that one of the planets has 1000 times the volume of the other. Thus, we conclude that Jupiter's volume must therefor be 1000 times that of the Earth, the same answer that we derived using Method 1!

**(Reader, pages 161 & 162)**

**87**

**Suppose the Earth was located twice as far from the Sun as it currently is. How would the gravitational force exerted by the Sun on the Earth at this new distance compare with its strength at its old distance?**

$$F_g = \frac{GM_1M_2}{R^2}$$

Suppose the Earth was located twice as far from the Sun as it currently is. How would the gravitational force exerted by the Sun on the Earth at this new distance compare with its strength at its old distance?

**87**

$$F_g = \frac{GM_1M_2}{R^2}$$

## Working With Ratios: "The Short Way" (from Mathematical Toolkit)

**Step 1: Write down the two quantities to be compared, and decide which one will be bigger.**

[redacted]

Suppose the Earth was located twice as far from the Sun as it currently is. How would the gravitational force exerted by the Sun on the Earth at this new distance compare with its strength at its old distance?

**87**

$$F_g = \frac{GM_1M_2}{R^2}$$

## Working With Ratios: "The Short Way" (from Mathematical Toolkit)

**Step 2: Write down the relevant formula as a proportionality by eliminating all constants (including variables that do not change).**

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## Working With Ratios: "The Short Way" (from Mathematical Toolkit)

**Step 3: Replace the variable(s) in Step 2 with the <u>factor(s)</u> by which the variable(s) is changing between the two situations being compared.**

[redacted]

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**87**

$$F_g = \frac{GM_1M_2}{R^2}$$

## Working With Ratios: "The Short Way" (from Mathematical Toolkit)

**Step 4: Use your answers from Steps 1 and 3 to answer the original question.**

[redacted]

Suppose the Earth was located twice as far from the Sun as it currently is. How would the gravitational force exerted by the Sun on the Earth at this new distance compare with its strength at its old distance?

**87**

$$F_g = \frac{GM_1M_2}{R^2}$$

Answer: If the Earth was located twice as far from the Sun as it currently is, the gravitational force exerted by the Sun on it would be only 1/4 (or, 25%) of the force currently exerted by the Sun on the Earth at its present location.

## Using Newton's Law of Gravity

**86**

Suppose the Earth was located twice as far from the Sun as it currently is. How would the gravitational force exerted by the Sun on the Earth at this new distance compare with its strength at its old distance?

$$F_g = \frac{GM_1M_2}{R^2}$$

→ Newton's gravity law is an example of an ***inverse square relation***: The force of gravity decreases as the inverse square of the distance separating the objects.

IF YOU DOUBLE THE DISTANCE, THE FORCE DECREASES BY A FACTOR OF 4
IF YOU HALVE THE DISTANCE, THE FORCE INCREASES BY A FACTOR OF 4

## Two Fundamental Questions about the Planets

**83**

What are the precise paths taken by the planets as they revolve around the Sun?

**Johannes Kepler (1571 - 1630)**

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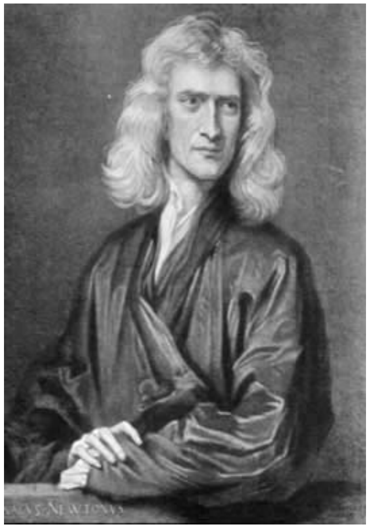

## Lecture 7: The Takehome Message

Isaac Newton discovered that the same force that causes an apple to fall to the Earth keeps the moon in its orbit around the Earth. He called this universal force

**gravity**.