Astronomy 101: A Mathematical Toolkit
Fall 2006, San Diego State University, Prof. D. Leonard

As described in the Course Syllabus handout, the mathematics that you will encounter in this course will be limited to algebra and geometry, of the sort typically found in a standard high-school curriculum. The purpose of this toolkit is to provide a quick “refresher” of the major mathematical concepts and techniques that will be used during the semester. Mastery of these skills will be extremely helpful throughout the course.

1. Algebra. Put simply, algebra means “solving for $x$”. You are given an equation containing numbers and a variable of undetermined value (i.e., $x$), and you are asked to determine the variable’s value. Here is a sample algebraic equation, and its solution.

$$2x = 6$$

Divide both sides by 2 to yield:

$$\frac{2x}{2} = \frac{6}{2}$$

Next, cancel the 2s on the top and bottom of the left hand side, and carry out the division on the right hand side to yield:

$$x = 3$$

A complete tutorial on the rules of algebraic manipulations can be found at:

http://www.mathleague.com/help/algebra/algebra.htm

(All web pages can be accessed through the “web links” section of the course homepage.) Please work through all of the examples given at this web site, but note that you do not need to consider the section on “sequences” for this class.

You will also find six nice animated examples of solved algebra problems at:

http://chemistry.boisestate.edu/rbanks/inorganic/algebra.html

2. Geometry. Formally, geometry is the mathematics of the properties, measurement, and relationships of points, lines, angles, surfaces, and solids. An excellent review of the basics of geometry that will be essential for this course is given at:

http://www.mathleague.com/help/geometry/geometry.htm

This web page is broken down into 6 categories. The first and second, “basic terms” and “angle terms” are essential to have under your belt, and should be rather familiar to you. In the “angle terms” section, the most useful relationship for this course will be the one derived for “corresponding angles”, i.e.:

For any pair of parallel lines 1 and 2, that are both intersected by a third line, such as line 3 in the diagram above, angle A and angle C are called corresponding angles. Corresponding angles have the same degree measurement. Angle B and angle D are also corresponding angles.
The third link, “figures and polygons” goes into some detail on a lot of different types of figures. It turns out that for astronomy, the most important shape by far is the circle. Thus, pay particular attention to the review of terms for the circle (e.g., the radius $r$). In the next section, “area and perimeter”, focus again on the circle, as you will be expected to know these relations:

(a) Circumference of circle $= 2\pi r$.
(b) Area of circle $= \pi r^2$.

In the next section, “Coordinates and similar figures”, you need only review the first section, entitled “What is a Coordinate?” This provides a very important review of graphing techniques, useful throughout the course. In the final section of the web page, entitled “Space figures and basic solids”, focus attention particularly on the sphere, for which:

(a) Surface area of sphere $= 4\pi r^2$.
(b) Volume of sphere $= \frac{4}{3}\pi r^3$.

Again, you are responsible for knowing these important relationships. Note that the key thing to remember in all of these formulas (for circles and spheres) is the power to which the radius $r$ is raised. This is what is really needed when determining the ratio of properties of circles and spheres (see below for an example).

3. Having completed the review of algebra and geometry, you are now ready to tackle the sheets attached to this handout, entitled “A Few Mathematical Skills”. In them, you will review powers of ten, scientific notation, unit conversions, the metric system, and ratios. Please pay particular attention to the section on ratios, as there will be particular emphasis on working with ratios throughout the term. Here’s an example of the use of ratios to compare properties of two objects. Note that these problems can be done without a calculator; indeed, you will not have access to a calculator during any exams, so it is important to get used to carrying out these sorts of problems on paper.

Sample question: Jupiter’s radius is roughly 10 times that of the Earth, and you may assume that both Jupiter and Earth are perfect spheres.

(a) What is the ratio of Jupiter’s circumference to that of the Earth?
   Solution: We start by defining what we know. Let’s call Earth’s radius $r_E$ and Jupiter’s radius $r_J$. Now, from the problem we are given that:
   
   $$r_J = 10r_E$$

   We also know that
   
   circumference $= 2\pi r$

   The ratio of Jupiter’s circumference to Earth’s circumference is therefore:
   
   $$\frac{\text{circumference of Jupiter}}{\text{circumference of Earth}} = \frac{2\pi r_J}{2\pi r_E}$$

   Notice that the $2\pi$ cancels from the top and the bottom of the equation, leaving just:

   $$\frac{\text{circumference of Jupiter}}{\text{circumference of Earth}} = \frac{r_J}{r_E}$$

   But, since $r_J = 10r_E$, we can substitute the expression $10r_E$ for $r_J$, which leaves us with:

   $$\frac{\text{circumference of Jupiter}}{\text{circumference of Earth}} = \frac{10r_E}{r_E}$$

   Canceling the $r_E$ from top and bottom, we find that:

   $$\frac{\text{circumference of Jupiter}}{\text{circumference of Earth}} = 10$$

   That is, Jupiter’s circumference is 10 times that of the Earth.
(b) What is the ratio of Jupiter’s volume to that of the Earth?

Solution: Again, we start by defining what we know, and call Earth’s radius $r_E$ and Jupiter’s radius $r_J$. And again, from the problem we are given that:

$$r_J = 10r_E$$

We also know that

$$\text{volume of sphere} = \frac{4}{3}\pi r^3$$

The ratio of Jupiter’s volume to Earth’s volume is therefore:

$$\frac{\text{volume of Jupiter}}{\text{volume of Earth}} = \frac{\frac{4}{3}\pi r_J^3}{\frac{4}{3}\pi r_E^3}$$

Notice that the $\frac{4}{3}\pi$ cancels from the top and the bottom of the equation, leaving just:

$$\frac{\text{volume of Jupiter}}{\text{volume of Earth}} = \frac{r_J^3}{r_E^3}$$

Since $r_J = 10r_E$, we can substitute $10r_E$ for $r_J$ in the above equation, which leaves us with:

$$\frac{\text{volume of Jupiter}}{\text{volume of Earth}} = \frac{(10r_E)^3}{r_E^3}$$

Now, since $(10r_E)^3 = 10^3r_E^3$, we have:

$$\frac{\text{volume of Jupiter}}{\text{volume of Earth}} = \frac{10^3r_E^3}{r_E^3}$$

Canceling the $r_E^3$ from top and bottom leaves us with:

$$\frac{\text{volume of Jupiter}}{\text{volume of Earth}} = 10^3$$

Since $10^3 = 1000$, we thus conclude that:

$$\frac{\text{volume of Jupiter}}{\text{volume of Earth}} = 1000$$

That is, Jupiter’s volume is 1000 times that of the Earth! Note that this is a much larger number than what we derived for the circumference ratio. This is due to the fact that while circumference scales as $r$, volume scales as $r^3$. Therefore, while the ratio of circumferences was only 10, the ratio of volumes is $10^3$, or 1000. Also, notice that at no time did these problems require us to know the actual value of Earth’s (or Jupiter’s) radius. All we needed was the ratio of the radii.

→If any of this mathematical “review” is difficult for you, please come seek help from me during office hours, or from the TAs during help room hours. The time to master the mathematics is now, at the start of the course!